**Homework 14**



**P12.1.6** Determine: (a) the effective inductance and effective capacitance in Figure P12.1.6 by applying: (i) the source absorption theorem, and (ii) KVL and comparing coefficients with those of a prototypical series *RLC* circuit; (b) *ω*0 and *α*;

(c) is the circuit, overdamped, underdamped, or critically damped?

**P12.1.6** (a) From KVL, *vSRC* = 4*vL* + 8*vC* + *vR* =

 + 100*i*, where the two capacitors in series are combined into a 2 μF capacitor. It is seen that the effective inductance is 4×10-2 ≡ 40 mH and the effective capacitance is 2/8 = 0.25 μF. The source absorption theorem is more conveniently applied in the frequency domain, where the dependent source is equivalent to an impedance 3**VL**/**I** + 7**VC**/**I**, where **VL** = *jωL***I** and **VC** = **I**/*jωC*. The equivalent impedance of the dependent source is therefore *jω*3*L* and 1/*jω*(*C*/7). In other words the equivalent inductance is 30 mH, which in series with 10 mH gives 40 mH, and the equivalent capacitance is 2/7 μF, which in series with 2 μF is 2(2/7)/(2 + 2/7) = 2/8 = 0.25 μF, as before.

(b)  krad/s; *α* = *R*/2*L* = 100/(0.08) =

1.25 krad/s.

(c) The circuit is underdamped.

**P12.1.10** The switch in Figure P12.1.10 is

opened at *t* = 0 after being closed for a long time. (a) Determine *iL*, *vC*, *dvC*/*dt*, and *diL*/*dt* just after the switch is opened. (b) Is the circuit reducible to a prototypical series *RLC* circuit

or a parallel *GCL* circuit?

**Solution:** (a) Just before *t* = 0, *iC* = 0 and *the* 3 mA divides between the 3 kΩ and 6 kΩ resistors, so that *IL*0 = 3×3/9 = 1 mA; *VC*0 = 6×1 = 6 V. These do not change on switching. Just after *t* = 0, 2*iC* + *iL* = 0, so that *iC* = -*iL*/2 = -0.5 mA = *C*, and **-1000 V/s. From KVL, *VC*0 + 1×*iC* = 6*IL*0 + *vL*. Hence, *vL* = 6 – 0.5 – 6 = -0.5 V = *L*, and **=** = -5 A/s.

(b) The circuit is not reducible to a prototypical second-order circuit.



**P12.2.19** The switch in Figure P12.2.19 is opened at *t* = 0 after being closed for a long time: (a) determine *R* for critical damping; using this value of *R*, determine: (b) *vC*(*t*), *t* ≥ 0+, *t* in ms; (c) the energy in μJ stored in the capacitor and inductor as functions of time.

**P12.2.19** (a)  rad/s. ,

*R* = 0.1×1000 = 100 Ω.

(b) *vC*(*t*) = ; at *t* = 0+, *vC*(0+) = *VSRC* = 2 = *A*; ; *iL*(0+) = , , which gives *B* = 0. Hence, , *t* ≥ 0+, *t* is in s, or, , *t* ≥ 0+,

*t* is in ms. The response is that of a first-order circuit.

(c) Energy stored in *C*:  μJ*, t* ≥ 0+, *t* is in ms, , , *t* is in s; energy stored in *L*: , , *t* ≥ 0+, *t* is in ms. The energy stored in the capacitor and inductor is the same.



**P12.2.20** The switch in Figure P12.2.20, is moved to position ‘b’ at *t* = 0, with zero initial energy storage. Determine: (a) for critical damping and (b) , , and  for +.

**Solution:** (a) For critical damping *R* = , where *ω*0 =   10 krad/s; *R* =  = 50 Ω. To have the parallel resistance equal to 50Ω, *ρ* = 0.

(b) Just before the switch is moved, *v* = 0 and *iL* = 0. Just after the switch is moved, these remain the same and *iC* = 0.1 A. For *t* ≥ 0, ; hence, *A* = 0. . It follows that V/s, or 100 V/ms. Hence, *v* = 100*te*-10*t* V, where *t* is in ms; *i*C = A, *t* being in seconds, or *iC* = 100(*e*-10*t* – 10*te*-10*t*) mA,

*t* being in ms. Since *iL* + *iC* + mA, *iL* = 100 – 100(*e*-10*t* – 10*te*-10*t*) mA, *t* being in ms, or *iL* = 100 –100(*e*-10*t* + 10*te*-10*t*) mA, *t* being in ms. As a check, A, with *t* in ms, or = mA. Integration gives the same answer.

**P12.2.22** The switch in Figure P12.2.22

is closed at *t* = 0 after being opened for a long time. Determine: (a) *iL*(0-), *iC*(0-), *iL*(0+),

*iC*(0+); (b) *iL*(*t*) for *t* ≥ 0+, assuming *R* = 1 Ω, *L* = 1 H, and *C* = 1 F.

**Solution:** (a) At *t* = 0-, the capacitor behaves as an open circuit and the inductor as a short circuit. It follows from KVL around the mesh of three *R*’s, that the voltage across the source is zero, so that *I*1 = 10 A and *IL*(0-) = 2I1 = 20 A. At *t* = 0+, *I*1 = 5 A, and 2*I*1 = 10 A. The current through the inductor and the voltage across the capacitor do not change. It follows that *iC* = -10 A.

(b) *ω*0 = 1 rad/s, and *α* = 1/2*RC* = 0.5 rad/s.  and V; at *t* = 0, *v* = 0, so that *A* = 0, and *v* = A. At *t* = 0, , which gives *B* =

-10/*ωdC*. Hence,  V, , and A. From KCL, *iL* + *iC* + *iR* = 10, so that *iL* = 10 – *iC* – *iR* = 10 +  +  = 10(1 + ). Substituting numerical values, *iL* = 10(1 + ) A.

**P12.3.9** Both switches in Figure P12.3.9 are closed at *t* = 0, with zero initial energy storage in the circuit. Determine *v*C(*t*) and *iL*(*t*) for

*t* ≥ 0+.

**Solution:** For *t* ≥ 0+, the circuit reduces to

that shown. *α* =  krad/s;  krad/s;  rad/s;  rad/s; ; at *t* = 0, *A* + *B* = -15,

*iL*(0+) = 0, so that -2*A* – 8*B* = 0; solving for *A* and *B* gives *A* = -20 and *B* = 5, so that  V, *t* is in ms. *i*(*t*) = *CdvC*/*dt* =  , *t* is in ms



**P12.3.17** Both switches in Figure P12.3.17 are moved at

*t* = 0 after being in their initial positions for a long

time. Determine *vC*(*t*) and *iL*(*t*) for *t* ≥ 0+.

**Solution:** At *t* = 0-, the 30 A source establishes initial conditions in the circuit. From current division, *iL*(0-) = 10 A and the current through the 4 Ω resistor is 20 A, so that

*vC*(0-) = 80 V. These do not change at *t* = 0+. As *t* → ∞, the capacitor acts as an open circuit and the inductor as a short circuit, so that *iL*(∞) = 0. The 5 A current flows through the 8 Ω resistor, making *vC*(∞) = 40 V.

When the 5 A source is set to zero, the circuit is a series circuit; *α* = *R*/2*L* = 8/20 = 0.4 rad/s;  rad/s; the responses are underdamped, and  rad/s.

 V; At *t* = 0+, 80 = *A* + 40, so that *A* = 40 V; at *t* = 0+, *iL* = -*CdvC*/*dt* = -*C*(-*αA* + *ωdB*) = 10; *ωdB* = -25 + 0.4×40 = -9, *B* =

-9/0.3 = -30. Hence, . *iL*(*t*) = -*CdvC*/*dt* 

 A.



**P12.3.18** The switch in Figure P12.3.18 is moved at *t* = 0 from position ‘a’ to position ‘b’ after being in position ‘a’ for a long time. Determine *v*C(*t*) and *iL*(*t*) for *t* ≥ 0+.

**Solution:** At *t* = 0-, the inductor behaves as a short circuit

and the capacitor as open circuit. *iL*(0-) = 6 A; The current in the upper

10 Ω resistor is 2 A, and the voltage across it is *vC*(0-) = 20 V. When the switch is moved, the source current is reduced to 3 A, so that *iL*(∞) = 3 A and *vC*(∞) = 10 V.

When the current source is set to zero, circuit reduced to a series *RLC* circuit having *R* = 20/3 Ω. Hence, *α* = *R*/2*L* = 5 rad/s and  rad/s. The responses are overdamped, with  rad/s and  rad/s.

 V; *vC*(0-) = 20 = *A* + *B* + 10, which makes *A* + *B* = 10. At *t* = 0+, 3 = *iL*(0+) + *CdvC*/*dt*, so that (1/6) *dvC*/*dt* = 1, or, *A* + 9*B* = 18; this gives *A* = 9 and *B* = 1. It follows that:  V, *t* is in s.

 A, *t* is in s.

**P12.3.19** The switch in Figure P12.3.19 is opened at

*t* = 0 after being closed for a long time. Determine *vC*(*t*) and *iL*(*t*) for *t* ≥ 0+.

**Solution:** At *t* = 0-, the inductor behaves as a short circuit and the capacitance as an open circuit. The resistance in series with *L* is (6||12) = 4 Ω. *iL*(0-) = (30 – 10)/4 = 5 A. From current division, the current in the 6 Ω resistor on the right is 5/3 A, so that *vC*(0-) = 10 + 6(5/3) = 20 V.

As *t* → ∞, with the capacitor behaves as an open circuit, *iL*(∞) = 0 and *vC*(∞) = 10 V. The circuit is a series circuit, with *R* = 4 Ω. Hence, *α* = *R*/2*L* = 4/0.8 = 5 rad/s and  rad/s.

The responses are critically damped.

 V; *vC*(0-) = 20 = *A* + *B* + 10, which makes *A* + *B* = 10. At *t* = 0+, *iL*(0+) = -*CdvC*/*dt*, so that -(0.1)(-5A + *B*) = 5, or, 5*A* – *B* = 50; this gives *A* = 10 and *B* = 0. It follows that:  V, t ≥ 0+ , *t* is in s

 A, *t* is in s.